An Extended Hybrid Markovian and Interval Unit Commitment Considering Renewable Generation Uncertainties

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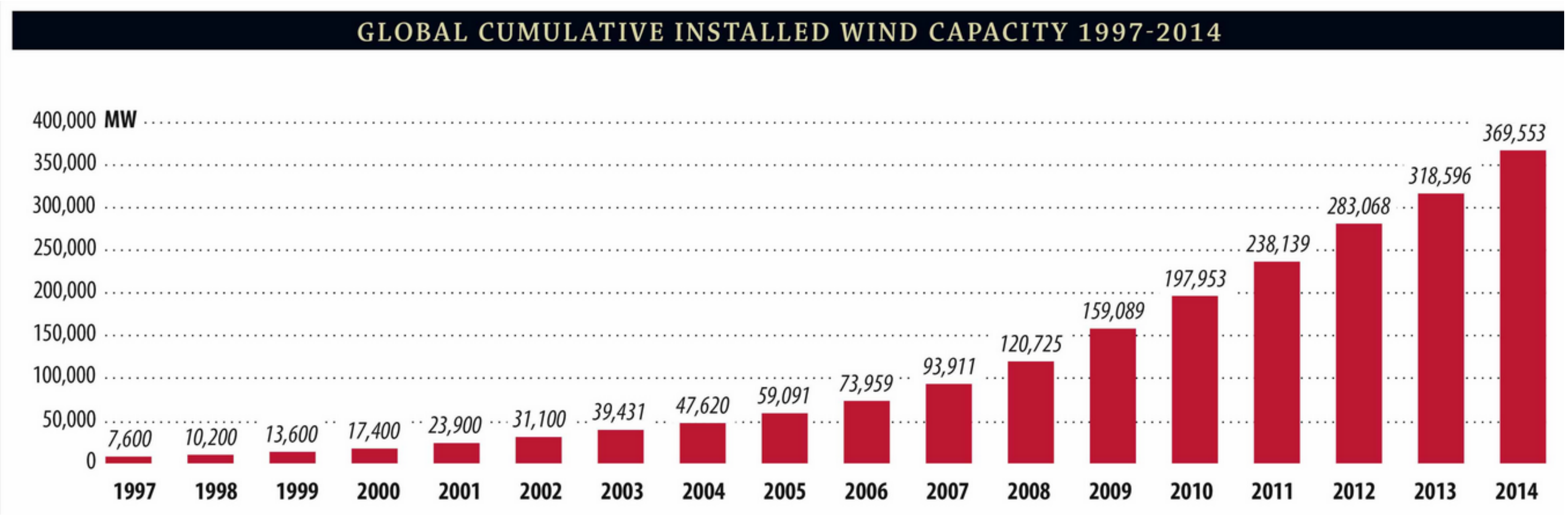
Introduction – Wind integration



[1]

- Is wind generation "free" beyond installation & maintenance?
  - Difficulties: Intermittent/uncertain nature of wind generation
    - In Spain, an unprecedented decrease in wind generation in Feb. 2012 is equivalent to the sudden down of 6 nuclear plants
    - 4 units not unusual ~ Hidden secret of intermittent renewables

1. http://breakingenergy.com/2015/03/19/wind-2000-gw-by-2030/

Existing Approaches

- **Deterministic Approach**
  - Uncertainties not explicitly considered
  - Solutions not robust against realizations of wind generation
  - Flexible ramping product is being investigated
- **Stochastic Programming**
  - Modeling wind generation by representative scenarios sampled from distributions
  - Solution methodology
    - Branch-and-cut
    - Benders' decomposition with branch-and-cut
    - Lagrangian relaxation with branch-and-cut
  - The number of scenarios: Too many or two few?

- **Robust optimization**
  - Uncertainties modeled by an uncertainty set, and the problem is optimized against the worst possible realization ~ Conservative
  - Min Max ~ Computationally challenging
  - Methodology: Benders' decomposition with outer approximation
- **Interval optimization** [2], [3], [4]
  - Wind generation modeled by closed intervals
  - Solutions to be feasible for extreme cases of system demand, transmission capacity, and ramp rate constraints ~ Conservative
  - Linear and efficient via interval arithmetic
  - Methodology: Benders' decomposition with branch-and-cut
- Better ways?

2. J. W. Chinneck and K. Ramadan, "Linear programming with interval coefficients," *Journal of the Operational Research Society*, Vol. 51, No. 2, pp. 209-220, 2000.
3. Y. Wang, Q. Xia, and C. Kang, "Unit commitment with volatile node injections by using interval optimization," *IEEE Transactions on Power Systems*, Vol. 26, No. 3, pp. 1705-1713, 2011.
4. L. Wu, M. Shahidehpour, and Z. Li, "Comparison of Scenario-Based and Interval Optimization Approaches to Stochastic SCUC," *IEEE Transactions on Power Systems*, Vol. 27, No. 2, pp. 913-921, 2012.

Outline

- Wind integration w/o transmission [5]
  - Stochastic UC formulation – Generation based on wind states
  - Problem solved by using branch-and-cut
- Wind integration considering transmission capacities [6]
  - Markovian and interval formulation – Generation based on local state
  - Numerical testing results via branch-and-cut
- An extended hybrid Markovian and interval approach (with the ABB team)
  - Generation of an isolated unit can depend on a remote wind farm
  - Solved by Surrogate Lagrangian Relaxation and branch-and-cut

5. P. B. Luh, Y. Yu, B. Zhang, E. Litvinov, T. Zheng, F. Zhao, J. Zhao and C. Wang, "Grid Integration of Intermittent Wind Generation: a Markovian Approach," *IEEE Transactions on Smart Grid*, Vol. 5, No. 2, March 2014.
6. Y. Yu, P. B. Luh, E. Litvinov, T. Zheng, J. Zhao and F. Zhao, "Grid Integration of Distributed Wind Generation: Hybrid Markovian and Interval Unit Commitment," *IEEE Trans. on Smart Grid*, early access since June 2015.

Stochastic Unit Commitment Formulation

- Modeling aggregate wind generation – A Markov chain
  - The state at a time instant summarizes the information of all the past in a probabilistic sense for reduced complexity
  - Net system demand = System demand – wind generation
- Minimize the sum of expected energy and startup/no-load costs

Exp. Energy cost   Start-up cost   No-load cost

$$\min_{\boxed{\{x_i(t)\}_{i,t},\{p_{i,n}(t)\}_{i,n,t}}} \sum_{i=1}^{I}\sum_{t=1}^{T}\left\{\underline{\sum_{n=1}^{N}\left[\boxed{\varphi_n(t)}C_{i,n}(p_{i,n}(t))\right]}+u_i(t)S_i+x_i(t)S_i^{NL}\right\}$$

  - s.t. system demand constraint for each state at every hour

$$\sum_{i=1}^{I} p_{i,n}(t) = P_n^D(t), \underline{\forall n,} \forall t$$

– Individual unit constraints

- Generation capacity constraints for each state

$$x_i(t)\,p_{i\min} \le p_{i,n}(t) \le x_i(t)\,p_{i\max}, \forall i, \forall t, \forall n$$

- Time-coupling ramp rate constraints for any state transition whose probability is nonzero

$$p_{i,m}(t-1) - \Delta_i \le p_{i,n}(t) \le p_{i,m}(t-1) + \Delta_i,$$

$$\forall i, \forall n, \forall t, \forall \underline{m \in \{m \mid \pi_{mn} \ne 0\}}$$ (Ramp-up and ramp-down)

- A linear mixed-integer optimization problem
- Solution methodology – Branch-and-cut

Difficulties when considering transmission

- Transmission capacities – A major complication
  - With congestion, wind generation cannot be aggregated
  - Global state: A combination of nodal states ~ Too many
- What can be done?
- Key ideas: Markov + interval-based optimization
  - Local states: Wind generation state at the local node
  - Divide the generation of a unit into two components
    - Local Markovian component: Depending on the local state
    - Interval component: To manage extreme combinations of non-local states
  - Less conservative as compared to pure interval optimization
  - Much simpler than pure Markov-based optimization

- Generation capacity constraints

The Markovian component: Depending on the local state n_i

$$x_{i,k}(t)\,p_{i,k}^{\min} \le \boxed{p_{i,k,n_i}^{M}(t)} + \boxed{p_{i,k,\bar{n}_i}^{I}(t)} \le x_{i,k}(t)\,p_{i,k}^{\max}, \forall i, \forall k, \forall t, \forall n_i, \forall \bar{n}_i$$

The interval component: Depending on the combination of non-local states $\bar{n}_i$

- Nodal injection

$$\boxed{P_{i,n_i,\bar{n}_i}(t)} = \underbrace{\sum_k p_{i,k,n_i}^{M}(t) + p_{i,n_i}^{W}(t) - p_i^{L}(t)}_{\text{Markovian nodal injection} \equiv P_{i,n_i}^{M}(t)} + \underbrace{\sum_k p_{i,k,\bar{n}_i}^{I}(t)}_{\text{Interval nodal injection} \equiv P_{i,\bar{n}_i}^{I}(t)}, \forall i, \forall t, \forall n_i, \forall \bar{n}_i$$

Markovian nodal injection $\equiv P_{i,n_i}^{M}(t)$ Interval nodal injection $\equiv P_{i,\bar{n}_i}^{I}(t)$

- System demand constraints ~ Sum of nodal injections = 0
  - Sum of nodal injections = 0 for both min/max guarantee the satisfaction for in-between demand levels

$$\sum_i P_{i,n_{i,\min},\bar{n}_{i,\min}}(t) = 0, \forall t \qquad \sum_i P_{i,n_{i,\max},\bar{n}_{i,\max}}(t) = 0, \forall t$$

- Transmission: |Power flow| ≤ Transmission capacity
  - A line flow depends on injections from many nodes and Generation Shift Factors (GSFs which can be + or -)

$$\boxed{f_l(t)=\sum_i (\underline{a_l^i}\cdot P_{i,n_i,\bar{n}_i}(t))}$$

Where are uncertainties?

$$=\sum_i\left[a_l^i\cdot\underbrace{\left(\sum_k p_{i,k,n_i}^M(t)+p_{i,n_i}^W(t)-p_i^L(t)\right)}_{\text{Markovian nodal injection } \equiv P_{i,n_i}^M(t)}\right]+\sum_i\left[a_l^i\cdot\underbrace{\left(\sum_k p_{i,k,\bar{n}_i}^I(t)\right)}_{\text{Interval nodal injection } \equiv P_{i,\bar{n}_i}^I(t)}\right],\ \forall l,\forall t$$

  - Determine extreme flows from wind uncertainties – contained in Markovian nodal injections – by considering signs of GSFs and extreme Markovian nodal injections

$$\sum_{i:a_l^i>0}[a_l^i\cdot\min_{n_i}P_{i,n_i}^M(t)]+\sum_{i:a_l^i<0}[a_l^i\cdot\max_{n_i}P_{i,n_i}^M(t)]\le\sum_i[a_l^i\cdot P_{i,n_i}^M(t)]$$

- Ramp rate constraints
  - For possible states, state transitions, and $p_{i,k,\bar{n}_{i,\min}}^I(t)$ and $p_{i,k,\bar{n}_{i,\max}}^I(t)$

- The objective function
  - With state probabilities and a few extreme realizations
  - Want to approximate the expected cost of all realizations w/o much complexity
    - Extremes only may not reflect the majority of realizations
    - Include a "typical realization" (e.g., the expected realization)
      - A set of deterministic constraints

$$\min \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{k=1}^{Ki} \left\{ \sum_{n_i=1}^{Ni} \left[w_{n_i,m_i}(t) C_{i,k}\left(p^{M}_{i,k,n_i}(t) + p^{I}_{i,k,m_i}(t)\right) \right.\right.$$
$$\left. + w_{n_i,M_i}(t) C_{i,k}\left(p^{M}_{i,k,n_i}(t) + p^{I}_{i,k,M_i}(t)\right)\right] \qquad (7)$$
$$\left. + \underline{w_E}(t) \boxed{C_{i,k}\left(p_{i,k,E}(t)\right)} + u_{i,k}(t) S_{i,k} + x_{i,k}(t) S^{NL}_{i,k} \right\}$$

Weight for the expected realization, adding up to 1

- Solution methodology – Branch-and-cut

Example 1 – IEEE 30-bus with 2 wind farms

- Data of two wind sites from April to September in 2006 [7]
  - Wind penetration level: 40%
    - W/o considering wind curtailment and load shedding
  - 1,000 Monte Carlo simulation runs

– Our approach provides 5.25% lower simulation cost than pure interval optimization

– Our approach is the most accurate in the sense of smallest APE*

– Trade-off: Solution robustness and conservativeness, modeling accuracy, and CPU time

Approach		Deter.	Interval	Ours
Optimi-zation	CPU time	2s	53s	1min53s
	Cost (k$)	248.659	280.672	253.403
UC cost (k$)		89.461	67.715	65.216
Simula-tion	E(Cost) (k$)	315.451	263.787	250.626
	APE	21.173%	6.401%	1.108%
	STD (k$)	74.058	33.117	34.613

Absolute percentage error* = |Optimization cost – simulation cost| / simulation cost × 100%

7. The National Renewable Energy Laboratory, Eastern Wind Dataset, 2010, [Online]. Available: http://www.nrel.gov/electricity/transmission/eastern_wind_methodology.html.

Outline

- Wind integration w/o transmission
- Wind integration with transmission capacity constraints
  - Can be conservative if a big unit does not have a local wind farm $\Rightarrow$ Interval Approach
- An extended hybrid Markovian and interval approach
  - Generation of an isolated unit can depend on a remote wind farm
  - Solved by a synergistic integration of Surrogate Lagrangian relaxation [8] and branch-and-cut [9]
  - Numerical testing results

8. M. A. Bragin, P. B. Luh, J. H. Yan, N. Yu, and G. A. Stern, "Convergence of the Surrogate Lagrangian Relaxation Method," *Journal of Optimization Theory and Applications*, Vol. 164, No. 1, January 2015, pp. 173-201.
9. M. A. Bragin, P. B. Luh, J. H. Yan, and G. A. Stern, "Novel Exploitation of Convex Hull Invariance for Solving Unit Commitment by Using Surrogate Lagrangian Relaxation and Branch-and-cut," to appear in *Proceedings of the IEEE Power and Energy Society 2015 General Meeting*, Denver, CO, USA

Key Ideas

- Allow an isolated unit to depend on a remote wind farm
  - Generation: A Markovian component + an interval component
- Modifications in the formulation?
  - System Demand
  - Ramp rates
  - Transmission capacity ~ Requiring the coordination of a isolated unit with a remote wind farm at a different bus

  $\Rightarrow$ More complicated

$\Rightarrow$ The Extended Formulation

– Simplified extreme Markovian flows – Can be conservative

$$\min f_l^M(t) = \sum_{\substack{i:a_l^i>0\\ i\neq k}} [a_l^i \cdot \min_{n_i} P_{i,n_i}^M(t)] + \sum_{\substack{i:a_l^i<0\\ i\neq k}} [a_l^i \cdot \max_{n_i} P_{i,n_i}^M(t)]$$

$$+ \sum_{k:a_l^k>0} [a_l^k \cdot \min_{n_k} P_{k,n_k}^M(t)] + \sum_{k:a_l^k<0} [a_l^k \cdot \max_{n_k} P_{k,n_k}^M(t)]$$

$$+ \sum_{j:a_l^j>0} [a_l^j \cdot \max_{n_k} P_{j,n_k}^M(t)] + \sum_{j:a_l^j<0} [a_l^j \cdot \min_{n_k} P_{j,n_k}^M(t)]$$

k: remote wind farms

j: linked units

n_k^* for nodes *k* and *j* can be different, but can be derived

– Interval flows

$$f_{l,c}^I(t) = \sum_i [a_l^i \cdot P_{i,c}^I(t)] + \sum_j [a_l^j \cdot P_{j,c}^I(t)]$$

f_l^I Interval flow has 2 possible combinations denoted as *c*

- How to solve the problem?

⇒ Decomposition and coordination of Lagrangian relaxation

- Lagrangian

$$L = \sum_{t=1}^{T}\{\sum_{i=1}^{I}[p_i(t)\cdot C_i + x_i(t)\cdot S_i^{NL} + u_i(t)\cdot S_i]$$

$$+\lambda(t)(\sum_i P_i) + \sum_l[\mu_{l,-}(t)(-f_l^{\max} - f_l(t))] + \sum_l[\mu_{l,+}(t)(f_l(t) - f_l^{\max})]\}$$

- Individual unit subproblems

$$\min_{\substack{x_i(t)\\ p_i(t)}} L,\ with\ L \equiv \sum_{t=1}^{T}\{[p_i(t)\cdot C_i + x_i(t)\cdot S_i^{NL} + u_i(t)\cdot S_i]$$

$$+\lambda(t)P_i + \sum_{l=1}^{L}\mu_{l,+}(t)(a_l^i\cdot P_i(t)) - \sum_{l=1}^{L}\mu_{l,-}(a_l^i\cdot P_i(t))\}$$

- Dual problem

$$\max_{\lambda,\mu}\Phi(\lambda,\mu),\ with\ \Phi(\lambda,\mu) \equiv \sum_{i=1}^{I}L_i^*(\lambda,\mu)$$

$$-\sum_{t=1}^{T}\sum_{l=1}^{L}(\mu_{l,+}(t) + \mu_{l,-}(t))f_l^{\max}$$

$$s.t.\ \mu_{l,+}(t) \geq 0, \mu_{l,-}(t) \geq 0$$

- Standard subgradient methods require L to be fully optimized
  - L is difficult to fully optimize
  - λ can suffer from zigzagging
  - Convergence proof and step size require q^*

Surrogate Lagrangian Relaxation

- Develop a new method, prove convergence, and guarantee practical implementability
  - Without fully optimizing the relaxed problem (s.t. the surrogate optimality condition) and without requiring q^*

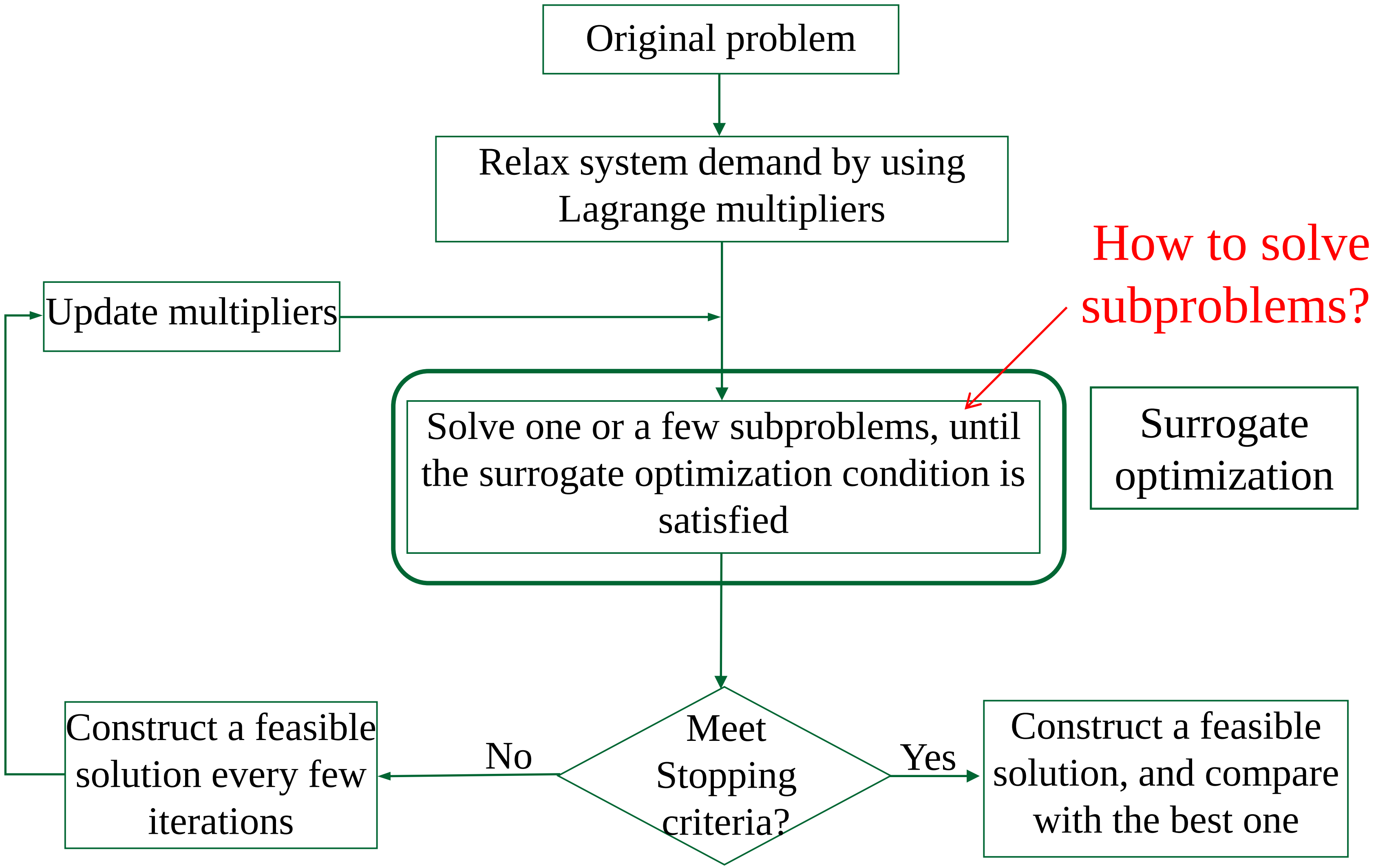
$$\lambda^{k+1} = \lambda^k + c^k \tilde{g}\left(x^k\right)$$

$$\left.\begin{array}{l} 1)\ c^k \sim \prod_{i=1}^{k} \alpha_i \to 0 \\ 2)\ \lim_{k\to\infty} \dfrac{1-\alpha_k}{c^k} = 0 \end{array}\right\}$$ Without requiring q^*!

- One possible example of α_k that satisfies conditions 1) and 2): $\alpha_k = 1 - \dfrac{1}{M \cdot k^p},\ 0 < p < 1,\ M > 1,\ k = 1, 2, \ldots$
- At convergence, the surrogate dual value approaches q^*
  ~ valid lower bound on the feasible cost

~ Overcomes all major difficulties of traditional LR

Schematic of Surrogate Lagrangian Relaxation



Difficulties of Standard Branch-and-Cut

- Branch-and-cut (B&C) can suffer from slow convergence because
  - Facet-defining cuts and even valid inequalities that cut areas outside the convex hull are problem-dependent and are frequently difficult to obtain
  - When facet-defining cuts are not available, a large number of branching operations will be performed
  - No “local” concept $\Rightarrow$ Constraints associated with one subproblem are treated as global constraints and affect the entire problem

Synergistic Combination with Branch-and-cut

- SLR relaxation and B&C are synergistically combined to simultaneously exploit separability and linearity:
  - Relax coupling constraints (system demand/transmission)
  - Solve a subproblem using branch-and-cut w/ warm start
    - The complexity is drastically reduced
  - Update multiplies by SLR – convergence w/o q^*
- Why is the new method effective?
  - Complexity of the algorithm is lower than that of B&C
  - Convex hulls for a subproblem do not change
  - Cuts for subproblems are effective
  - Feasible solutions can be effectively obtained

- subproblem

$$L = \sum_{i=1}^{I} \boxed{\left\{ \sum_{t=1}^{T} \left(C_i(p_i(t), t) + S_i(t) - \lambda(t) p_i(t) \right) \right\}} + \sum_{t=1}^{T} \lambda(t) P_d(t)$$

Implementation of SLR + Branch-and-Cut

- Testing system – IEEE 30-bus 41-branch 24-period
  - Relax all coupling system demand and transmission capacity constraints
  - Form individual unit subproblems s.t. unit-wise constraints
  - Configurations: 10 wind farms, 10 co-located units, 2 non-colocated cheap units
- Implementation – In CPLEX 12.6.0.0 on Dell Precision M4500
  - SLR implemented using ILOG Script for OPL
    - Flow control, load data, generate models, update multipliers, warm start …
  - Subproblems solved by the CPLEX using branch-and-cut
  - Multipliers are initialized according to priority list
    - System marginal costs for extreme and expected system demands timed the weights as those in the objective function

Units' characteristics

Unit #	pmin	pmax	Offer price	Start-up cost	Associated wind farm
Co-located units					
1	5	157	62.6	786.8	1
2	8	100	56.7	945.6	2
3	14	157	62.6	700	3
4	22	100	56.7	800	4
5	10	60	42.1	1000	5
6	3	157	62.6	850	6
7	15	100	56.7	950	7
8	10	80	41.1	1243.5	8
9	5	157	62.6	600	9
10	25	100	56.7	750	10
Non-co-located units					
11	10	80	37.2	900	2
12	10	90	39	1000	8

Wind farms' characteristics

- All wind farms are assumed to be identical for each level of wind penetration

Wind penetration level	Pmax for wind farm
5%	4 MW
15%	12 MW
25%	20 MW

- A penalty of $5000/MWh on wind curtailment is incurred beyond a certain threshold
  - For example, for the 25% case, if 10 MW out of 20 MW available are not used, then penalty is incurred

Testing results

- Consider 5% wind penetration

		Non-extended case		Extended case	
Method		SLR+B&C	B&C	SLR+B&C	B&C
Lower bound (k$)		292,508.74	294516.13	291,740	295328.95
Feasible cost (k$)		314,411	N/A**	316,478	N/A
Gap		6.96%	N/A	7.92%	N/A
Clock time* (s)	Iterations	189	1200	310	1200
	Heuristics	231		110	
Wind Curtailment (k$)		0	N/A	0	N/A
Load Shedding (k$)		656.49	N/A	688.17	N/A

Clock time* : solving time + other time (13 iterations)
**: B&C cannot solve because of shortage of power from conventional generators

$$\alpha_k = 1 - \frac{1}{M \cdot k^p},\ p = 1 - \frac{1}{k^r},\ r = 0.1,\ M = 30,\ k = 1, 2, \ldots$$

Testing results

- Consider 15% wind penetration

		Non-extended case		Extended case	
Method		SLR+B&C	B&C	SLR+B&C	B&C
Lower bound (k$)		268,975	265,020.46	269,617	N/A**
Feasible cost (k$)		284,455	331,835.67	283,619	N/A
Gap		5.44%	20.14%	4.93%	N/A
Clock time* (s)	Iterations	288	1200	257	N/A
	Heuristics	12		43	
Wind Curtailment (k$)		0	0	0	N/A
Load Shedding (k$)		6,376.07	1,243.68	3,522.8	N/A

Clock time* : solving time + other time (16 iterations)
**: CPLEX was out of memory and computer froze

$$\alpha_k = 1 - \frac{1}{M \cdot k^p},\ p = 1 - \frac{1}{k^r},\ r = 0.1,\ M = 30,\ k = 1, 2, \ldots$$

Testing results

- Consider 25% wind penetration

<table>
<tr><th colspan="2"></th><th colspan="2">Non-extended case</th><th colspan="3">Extended case</th></tr>
<tr><td colspan="2">Method</td><td>SLR+B&C</td><td>B&C</td><td>SLR+B&C</td><td>B&C</td><td>B&C</td></tr>
<tr><td colspan="2">Lower bound (k$)</td><td>266,304</td><td>250,447.8</td><td>244,120</td><td>241,892.04</td><td>241,997</td></tr>
<tr><td colspan="2">Feasible cost (k$)</td><td>267,379</td><td>312,028.4</td><td>258,026</td><td>1,766,826.7</td><td>253,726</td></tr>
<tr><td colspan="2">Gap</td><td>0.4%</td><td>19.73%</td><td>5.83%</td><td>86.31%</td><td>4.62%</td></tr>
<tr><td rowspan="2">Clock time* (s)</td><td>Iterations</td><td>290</td><td rowspan="2">1,200</td><td>720</td><td rowspan="2">3,600
(1 hour)</td><td rowspan="2">12,890
(3h35min)</td></tr>
<tr><td>Heuristics</td><td>10</td><td>480</td></tr>
<tr><td colspan="2">Wind Curtailment (k$)</td><td>0</td><td>0</td><td>25.3105</td><td></td><td>0.04</td></tr>
<tr><td colspan="2">Load Shedding (k$)</td><td>4,151.33</td><td>2,522.75</td><td>2,857.89</td><td></td><td>1,074.4</td></tr>
</table>

Clock time* : solving time + other time (16 iterations)

$$\alpha_k = 1 - \frac{1}{M \cdot k^p},\ p = 1 - \frac{1}{k^r},\ r = 0.1,\ M = 30,\ k = 1, 2, \ldots$$

Conclusion

- An important but difficult issue with no practical solutions
- A major breakthrough for effective grid integration of intermittent wind and solar, with key innovations:
  - Markov processes as opposed to scenarios to model wind generation for reduced complexity
  - Markov + interval-based optimization to overcome the complexity caused by transmission capacity constraints
  - The extended approach further reduces the conservativeness
- Opens a new and effective way to address stochastic problems w/o scenario analysis or over conservativeness
- The innovative SLR + B&C opens a new direction on solving large mixed-integer linear programming problems

Thank You!